

Fractional operators, Lévy stable distributions and Bessel polynomials

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ABSTRACT

We introduce and study an extension of the heat equation relevant to relativistic energy formula involving square root of differential operators. We furnish exact solutions of corresponding Cauchy (initial) problem using the operator formalism invoking one-sided Lévy stable distributions. We note a natural appearance of Bessel polynomials which allow one the obtention of closed form solutions for a number of initial conditions. The resulting relativistic diffusion is slower than the non-relativistic one, although it still can be termed a normal one. Its detailed statistical characterisation is presented in terms of exact evaluation of arbitrary moments and is compared with the non-relativistic case.