Bethe Ansatz and integrability

TADEUSZ LULEK[1]

[1]*Faculty of Physics, Adam Mickiewicz University, Umultowska 85, Poznań*
tadlulek@amu.edu.com

ABSTRACT

Bethe Ansatz (BA) is known as a genious mathematical guess which admits an exact solution to some non-trivial many-body problems (integrable systems) which include the eigenproblem of the Heisenberg Hamiltonian for the ring of *N* spins ½ with isotropic nearest-neighbour interaction (the XXX model) sa the seminal example. Algebraic Bethe Ansatz (ABA) of Faddeev and Tachtajan is a formalism which reproduces this guess in an elegant algebraic form, in terms of monodromy, Lax operators and transfer matrix. Both approaches, the original BA (known nowadays as the coordinate BA) and ABA, imply a need to solve the system of BA equations for spectral Bethe parameters – the system of r highly nonlinear algebraic equations, with *r* being the number of spin deviations from the ferromagnetic saturation state |0>. Then, each regular solution (λ_1, …, λ_r) yields the corresponding exact eigenstate of the system in a form $B(\lambda_1)...B(\lambda_r)$ |0>, where $B(\lambda)$ is the creation operator for an excitation characterized by the spectral parameter λ. The aim of our report is to point out another way for determination of exact eigenstates within ABA formalism, namely – by application of the set of *N* commuting observables $T^{(l)}$ (constants of motion) from the transfer matrix $T(\lambda)$. Knowledge of spectra of these constants of motion allows us to determine exact eigenstates of the Heisenberg Hamiltonian, along the standard quantum-mechanical prescription of Dirac. It is worth to mention that the numerical problem of solving non-linear BA equations is replaced by elementary algebraic manipulations with matrices of finite dimension, and the results are given in the form of density matrices – the one-dimensional projectors onto exact eigenstates. Bethe parameters can be then recovered from already known eigenstates by inverse BA.