Some physical interpretations of Galois symmetries of exact Bethe Ansatz eigenstates

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ABSTRACT

We consider Galois symmetries of exact Bethe Ansatz solutions for short magnetic Heisenberg rings of the spin 1/2 within the XXX model.

For the pentagonal ring ($N = 5$ nodes), eigenstates can be expressed within the cyclotomic extension $\mathbb{Q}(\omega)$, $\omega = \exp(2\pi i/5)$, of the prime field $\mathbb{Q}$ of rational numbers. These eigenstates, however, do not exhibit the form of Bethe Ansatz (BA), and transformation to this form requires further extension of $\mathbb{Q}(\omega)$ by the Bethe number field $\mathbb{B}$ generated by spectral parameters of constituents of strings, which enter BA solutions. On the other hand, complex Heisenberg number field (the minimal field sufficient to express explicitly all BA eigenstates), coincides with the cyclotomic field $\mathbb{Q}(\omega)$. We point out some similarities of the extension mentioned above, to crystallographic constructions, and propose appropriate interpretation within the midst of string configurations, rigged by quasimomenta.

For the heptagonal ring ($N = 7$ nodes) we focus on the problem of a double degeneracy at the center of the Brillouin zone in the three-magnon sector. Such a degeneracy admits an arbitrary choice of basis within the doublet being considered. We point out, however, that Bethe Ansatz solutions impose a definite choice of a basis, with exactly prescribed spectral parameters. We demonstrate, that rigging of common eigenstates by appropriate quasimomenta from the Brillouin zone is fully consistent with both dynamics, and parity symmetry.